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Pinning and Long-Time-Scale Behavior in Traveling-Wave Convection

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We study nonlinear traveling-wave (TW) and stationary states of convection in experiments in ethanol–water mixtures. While the TW phase velocity as a function of Rayleigh number has been recently shown to be in agreement with the predictions of theory and numerical calculations, we find that this velocity is temporally modulated at frequencies corresponding to the travel time of a single convection roll and of a roll pair past a point stationary in the convection cell. This modulation could be due to the pinning of the convection pattern by experimental inhomogeneities. For large Rayleigh numbers where stationary overturning convection is expected, we sometimes observe extremely slow unidirectional TW states. For larger Rayleigh numbers, this slow TW state starts and stops intermittently on a characteristic time scale of several days. The possible origin of these phenomena and their potential utility are discussed.

KEY WORDS: Convection; traveling waves; pattern formation and dynamics; binary fluid mixtures.

1. INTRODUCTION

Convection in binary-fluid mixtures has been extensively studied recently as a simple model system exhibiting complex spatiotemporal behavior. Many experimental and theoretical studies have focused on the dynamics of these convective states at or near the onset of convection (see, e.g., ref. 1). Some of the observed states and their dynamics have been explained by weakly nonlinear theory based on the Ginzburg–Landau amplitude

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equations.[2] More recently, there have been attempts to characterize the large-amplitude, spatially-uniform nonlinear states by perturbation analysis around the pure-fluid convecting state[3] and by numerical simulations.[4] Experimental verification[5] of many of the predictions indicate that we now have an understanding of the simplest fully nonlinear TW state in this system.

In this paper, we describe experimental behavior which is beyond the scope of these predictions. In particular, the TW phase velocity of the uniform convection state is not constant, but is modulated in time at both the TW frequency and twice the TW frequency. In addition, for large Rayleigh numbers at which stationary overturning convection (SOC) is expected, a very slow unidirectional TW state is sometimes observed. This slow propagation can vary intermittently in direction and magnitude on very long time scales. The most likely cause of these observations is a combination of pinning and lateral currents in the convection cell. If this is the case, it is possible that one could exploit these phenomena to study the dynamics of waves moving past purposely introduced pinning centers in this nonequilibrium system.

Convection in pure fluids can be characterized by the usual two dimensionless parameters: the Prandtl number P and the Rayleigh number R, which is proportional to the applied temperature difference. For Rayleigh numbers above a threshold value R_c ($R_c \approx 1708$ for no-slip boundaries), convection begins as the thermally driven buoyancy overcomes viscous and thermal dissipation. The description of convection in fluid mixtures which exhibit the Soret effect, such as alcohol and water, requires two additional parameters: the Lewis number L, which is the ratio of mass diffusion to thermal diffusion, and the separation ratio ψ, which is the ratio of density changes due to concentration to those due to temperature. In this paper, we consider the regime where $\psi < 0$, i.e., the Soret effect causes the alcohol to diffuse toward the colder region, and the conducting state is stabilized by the resulting linear concentration gradient. Thus, the onset of convection occurs at a higher Rayleigh number than the pure-fluid case, i.e., $r_{c0} \equiv R/R_c > 1$ at onset. Furthermore, linear stability analysis predicts that for $\psi < -L^2$ ($L \sim 10^{-2}$), conduction should give way to an oscillatory convective state at onset.[1]

Experimentally, one observes that the linear oscillatory state takes the form of traveling waves which grow in amplitude at onset until they trigger a subcritical Hopf bifurcation to a large-amplitude, slow-TW state. This nonlinear TW state was recently studied both by perturbation analysis in small ψ around states of pure-fluid convection[3] and by direct finite-difference numerical simulation of the fluid equations.[4] In these studies, the linear, Soret-driven vertical concentration gradient in the conduction state, which provides the restoring force for the linear oscillatory instability, is eliminated in the interior of the fluid by convective mixing. Thus, in these large-amplitude nonlinear TW states, vertical concentration gradients remain only in thin horizontal boundary layers at the top and bottom of the fluid layer, and small lateral concentration gradients are advected into the boundaries between the convection rolls. These lateral concentration gradients cause the rolls to travel at phase speeds much lower than the linear Hopf frequency observed at onset. Along the nonlinear traveling-wave branch, the TW velocity is predicted to vary as a function of Rayleigh number. At a reduced Rayleigh number r^* the convection rolls stop, and pure-fluid-like stationary convection results. This picture of the nonlinear TW state has been confirmed by recent experiments.[5] Good agreement was obtained for the functional dependence of TW velocity on r over the entire TW branch.

2. EXPERIMENTAL APPARATUS

The experiments are conducted in an annular cell with rectangular cross section. The apparatus is the same as described in ref. 5. The working fluid is contained in a cell with a rhodium-plated, mirror-polished copper bottom plate, a sapphire top plate, and plastic sidewalls (ULTEM polyetherimide). The top-plate temperature is set to 25.00°C and regulated to ± 0.7 mK by water flowing azimuthally over its upper surface. The bottom plate is heated electrically to a temperature of 29.5–31.0°C with a similar regulation. The vertical height of the cell is $d = 0.309$ cm, and the radial spacing is $1.29d$, forming a channel of mean length $67.1d$.

The working fluid is an 8.00% by weight mixture of ethanol in water. At an average temperature of 27.5°C, the fluid parameters are[6] $\psi = -0.26$, $P = 9.2$, and $L = 0.008$. The vertical thermal diffusion time is $\tau_v = d^2/\kappa = 74.2$ sec, where κ is the thermal conductivity of the mixture. Rayleigh numbers are normalized by the critical Rayleigh number for the onset of convection in a pure fluid with the same properties as this mixture. Shadowgraph visualization from above[7] is used to observe the convection rolls, which align radially and propagate azimuthally when traveling. The pattern is recorded by an annular CCD array of 720 pixels spanning the annular width of the cell and is digitized by computer. The positions of the peaks in shadowgraph intensity, which correspond to cold downflow regions between convection roll pairs, are computed versus time for each convection state. As an example, the peak positions for a state of uniform convection at $r = 1.956$ are shown in Fig. 1. At each time step, the average spatial position of the entire pattern is determined, and then the spatially-averaged velocity as a function of time is calculated. We define the phase

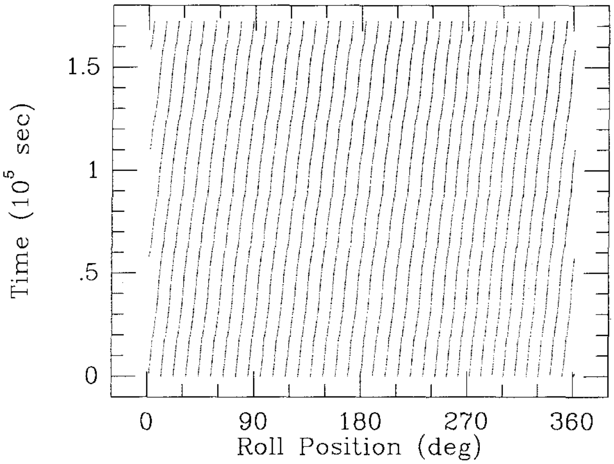

Fig. 1. Position of peaks in shadowgraph intensity (downflow roll boundaries) as a function of time for a uniform state of TW convection with wavenumber $k = 3.28$ at $r = 1.956$.

velocity of the nonlinear convection pattern as this spatially-averaged velocity.

In Fig. 2, taken from ref. 5, the time-averaged (mean) phase velocity measured as a function of Rayleigh number is shown. Also shown are least-squares fits of the theory (solid line) and numerical calculations (dashed

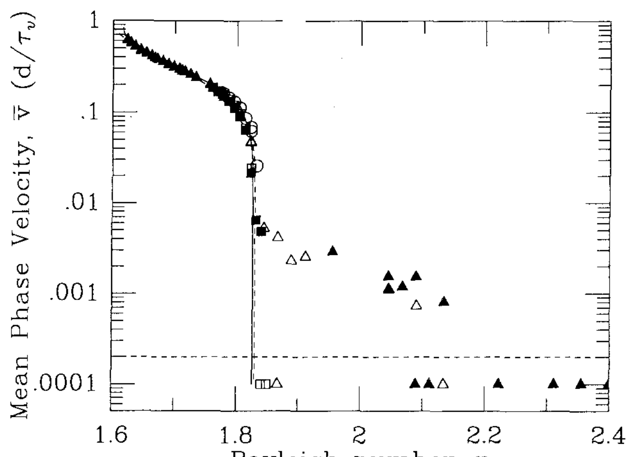

Fig. 2. Time-averaged, uniform convection phase velocity $\bar{v}$ vs. Rayleigh number, from ref. 5. The horizontal dashed line indicates the limits of our resolution; the points below are regarded as stationary convection. Filled (open) symbols indicate decreasing (increasing) r. Triangles denote positive-direction TWs with $k = 3.28$; squares, negative with $k = 3.28$; circles, negative with $k = 3.18$. The solid (dashed) line is the fit to perturbation theory (numerical calculations) of ref. 3 (ref. 4).

line) to the data in which the free parameters are r^* and the scale along either axis. As discussed in detail in ref. 5, the functional form of the data below $r^* = 1.824$ is in good agreement with predictions. Above r^*, very slow TW states are sometimes observed which, in contrast to those below r^*, have a preferential direction. We discuss these states in more detail below.

3. MODULATED TW SPEEDS

The slope of the space-time lines in Fig. 1 is not constant, indicating that the velocity of the rolls is temporally modulated, as shown in Fig. 3. For illustration, a state with a particularly large relative variation in velocity was chosen. The velocity oscillates about the mean with periods equal to the travel times of a single convection roll and of a roll pair past a fixed point. In Fig. 4 the amplitude spectrum shows that the dominant peaks are near $1/\bar{\tau}$ and $2/\bar{\tau}$, where $\bar{\tau}$ is the mean period for passage of a roll pair. The TW speed oscillates in a similar way at all Rayleigh numbers along the entire TW branch both above and below r^*. In Fig. 5 the rms size of the modulation Δv is shown along with the mean TW speed as a function of Rayleigh number. The magnitude of the modulation has significant scatter, but appears to be continuous, with perhaps a change in slope at r^*. Thus, the speed modulation appears to be independent of the basic driving mechanism of the traveling waves. The relative speed modulation

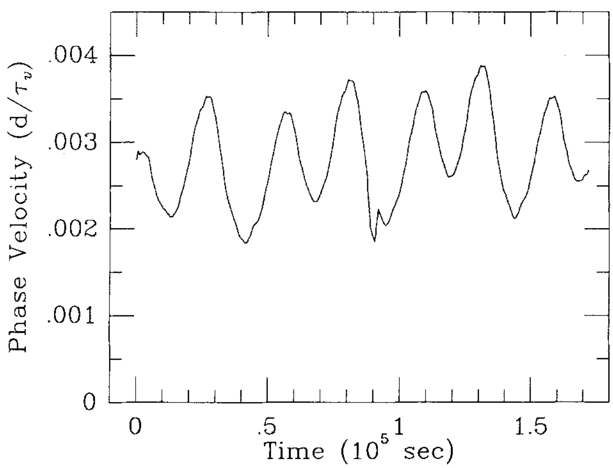

Fig. 3. Spatially-averaged TW phase velocity as a function of time for the state in Fig. 1. The mean period $\bar{\tau}$ for a roll pair to pass a stationary point in the cell is about 5×10^4 sec.

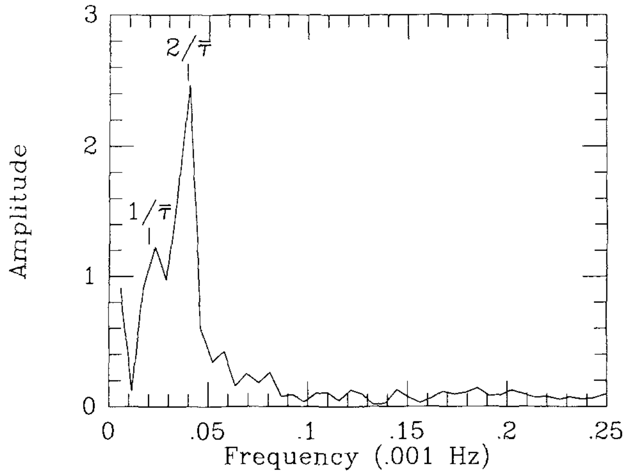

Fig. 4. Frequency spectrum of the phase velocity (in arbitrary units, proportional to the square root of the power spectral density) for the state in Figs. 1 and 3. The time-averaged frequencies corresponding to the passage of a roll pair ($1/\bar{\tau}$) and of a roll ($2/\bar{\tau}$) past a fixed point are shown.

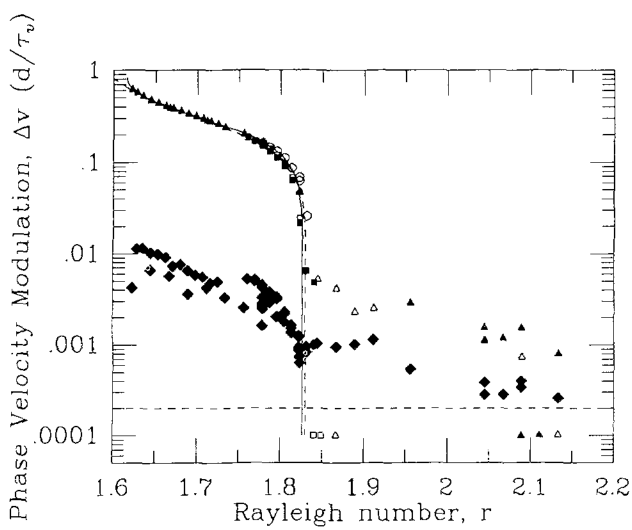

Fig. 5. The solid diamonds represent the rms variation of TW speed Δv vs. Rayleigh number for the states in Fig. 2. The smaller symbols are identical to those of Fig. 2.

$\Delta v/\bar{v}$ is shown in Fig. 6. In the two regimes on either side of r^*, the relative modulation is about constant. The two constant magnitudes are quite different, reflecting the differences in $\bar{v}$ along with the smoothly varying Δv.

A geometrical or thermal experimental inhomogeneity which results in a thermal or frictional pinning center of the convection pattern is consistent with these observations. Such a pinning center would have to couple to each convection roll as well as to roll pairs to produce the observed modulation. For example, a simple local hot-spot in the bottom plate does not work (at least if the coupling is linear), since it would only give a modulation period commensurate with the passing of up-flow boundaries between traveling convection roll pairs and not at the frequency $2/\bar{\tau}$.

We were not able to locate such a pinning center in our experiments. The wavelengths of the roll pattern varied by a maximum of $\pm 5\%$ around the cell, but the pattern is exactly repeatable as the rolls propagate. Each successive roll pair has the same wavelength at a given point in the annulus, even though this wavelength is observed to vary on distances comparable to the cell height. Thus, the variation is likely to be due to optical distortions instead of variations in roll spacing due to a pinning center. An absolute distance scale in the annulus, such as radial calibration lines on the mirror, would be needed to separate optical effects from actual wavelength changes, and this was not done.

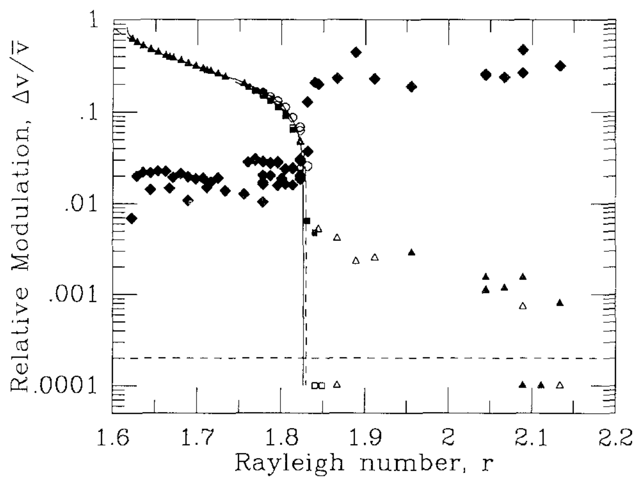

Fig. 6. Relative speed variation $\Delta v/\bar{v}$ vs. Rayleigh number for the states in Fig. 2. The smaller symbols are identical to those of Fig. 2.

4. ANOMALOUS PROPAGATION FOR $r > r^*$

The time-averaged TW speed $\bar{v}$ for uniform traveling-wave convection is shown as a function of r in Fig. 2, taken from ref. 5. For $r < r^*$, the fits to theory and calculations are quite good, but for $r > r^*$, we sometimes find an anomalous velocity instead of stationary overturning convection (SOC) as expected. An example of this very slow TW is shown in Fig. 1 for $r = 1.956$ ($r^* = 1.824$). The slow TW state for $r > r^*$ was only observed in one direction, which we arbitrarily designate as positive (triangles in Fig. 2), except for one instance in which very slow waves started from SOC upon lowering r very near r^* (the two filled squares near $r = 1.84$, $\bar{v} = 0.005d/\tau_v$). No other slow TW states in the negative direction were observed for $r > r^*$, whether the state was obtained from SOC by lowering r or from usual TW by raising r. In fact, in one experimental run as r was raised from just below to just above r^*, a negative-direction TW stopped and reversed direction to give the slow TW state in the usual positive direction.

The question arises as to what drives the very slow, unidirectional TWs observed above r^*. The mechanism of gradients of concentration in thin boundary layers[3,4] driving these slow TWs does not explain the data, since this effect is symmetric in direction and predicts stationary convection above r^*. It would be consistent with most of the data to suppose that there is an extraneous fluid current in our apparatus which drives the slow unidirectional TWs in the SOC regime. A simple variation in d due to tilt of the top or bottom plates or variation in annulus width due to the plastic ring and disk being off-center does not give a preferred direction. A possible scenario involves the very narrow slots in the bottom of the plastic sidewalls which are used to fill the cell. If these slots are not exactly diametrically opposed, then a current in one and out the other might give a preferred direction. The origin of such a current is unknown. In any case, the unidirectional nature of these states and the small variation of speed with Rayleigh number compared to the nonlinear TW branch, $r < r^*$, indicates that some different physics causes the convection rolls to translate.

In the anomalous propagation regime, $r > r^*$, the modulation of the very slow waves can be of the same size as the mean velocity, leading to interesting, long-time-scale behavior. Near $r \approx 2.1$, where the convection becomes absolutely stationary, we observed states which may be intermittently pinned. In Fig. 7 the convection pattern is alternately stationary and moving very slowly. The time series spans about 2 weeks, and during this time the convection rolls were seen to remain stationary and then move either one or two wavelengths before becoming stationary again.

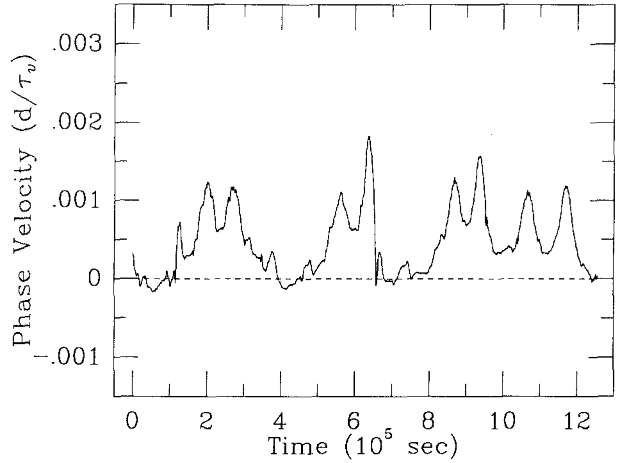

Fig. 7. Spatially-averaged phase velocity as a function of time for a state of uniform convection with $k = 3.28$ at $r = 2.089$.

When traveling, the pattern has a mean phase speed $\bar{v} \approx 7.3 \times 10^{-4} d/\tau_v$, corresponding to a mean period $\bar{\tau} \approx 2 \times 10^5$ sec for the passage of a roll pair. In Fig. 8 over a similar length of time, the TWs gradually slowed down, suddenly reversed direction, and then perhaps started the process over again. In this case, the average period $\bar{\tau}$ increased from about 0.8×10^5 sec near $t = 0$ to about 1.3×10^5 sec near $t = 4 \times 10^5$ sec, where t is the time plotted in Fig. 8. These examples shown in Figs. 7 and 8 could indicate that

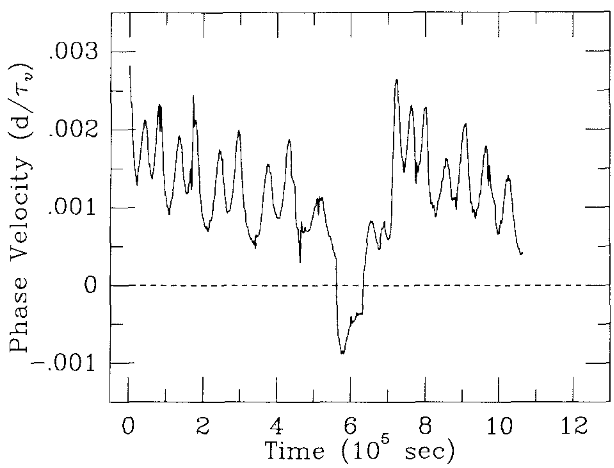

Fig. 8. Spatially-averaged phase velocity as a function of time for a state of uniform convection with $k = 3.28$ at $r = 2.045$.

the driving of the TWs for $r > r^*$ is erratic or that complex interactions may take place as a result of the combination of the driving and pinning forces.

5. CONCLUDING REMARKS

We have studied the dynamics of quasi-one-dimensional patterns in convection in an ethanol–water mixture in a narrow annular channel. The focus of this paper is on observed effects which appear to be beyond those expected for a uniform system. In particular, we observe very slow unidirectional translations of the pattern and modulations in the phase velocity at the characteristic frequencies corresponding to the passage of a roll and a roll pair by a point fixed in the cell. It may be possible to exploit these apparently extrinsic effects to study the pinning of waves in non-equilibrium systems.

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